Capital Structure under Changing Uncertainty

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Abstract

We explain the puzzle of falling firm leverage during the period of decreasing aggregate volatility from 1970 – 2010 with a simultaneous increase in firm level volatility and varying degrees of firm access to public debt markets. By incorporating friction between public and private debt, a structural model where firms make financing policy decisions in presence of uncertainty can explain a significant amount of these puzzling trends. The model results are consistent with the trend that the declining firm leverage is driven by unrated firms. This finding relates to the established trends of increasing idiosyncratic volatility and decreasing macroeconomic volatility. Firms without access to public debt markets have to insure themselves against increasingly volatile idiosyncratic shocks through conservative financing decisions. The model and the empirical results demonstrate that for firms with access to public debt markets, the cost of financing is more closely connected with macroeconomic conditions.

Keywords: Firm-Level Volatility, Aggregate Volatility, Firm Leverage.

JEL Classification: G32, D22, D24, E32.
I Introduction

Over the past 40 years, firm leverage has been decreasing (See Figure 1). At the same time, there has been a decrease in aggregate volatility of the economy which leads to a reduction in financing cost of firms.\(^1\) Considered together, these two trends pose a puzzle – firms are reducing leverage even though the cost of external financing is falling. Through a structural model where firms make financing policy decisions in presence of uncertainty, our paper seeks to explain these puzzling trends.

\begin{center}
\includegraphics[width=\textwidth]{figure1.png}
\end{center}

Figure 1: Leverage of US Firms Over Time

Two important observations deserve attention before we present our mechanism. (i) We document that firms over time have chosen dramatically different capital structure policies based on the source of financing.\(^2\) The decline in leverage is much more pronounced among

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\(^1\)See Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) for example for documentation of decrease in aggregate volatility.

\(^2\)Faulkender and Petersen (2006) show that firms that have access to the public bond markets, as measured
firms that do not have access to public debt markets (See Figure 2). (ii) As noted above, while aggregate volatility has been declining, literature has also shown that firm level volatility has in fact been increasing.³

![Figure 2: Leverage of US Firms by Rating](image)

In this paper, we argue that a substantial fraction of the pattern in firm leverage can be explained through precautionary financing motives of firms, which are insuring themselves against increasing idiosyncratic volatility. We suggest the following mechanism. We argue that trends in aggregate and idiosyncratic volatility do not affect all firms equally. Firms with a debt rating, have significantly more leverage. Although firms with a rating are fundamentally different, these differences do not explain their findings. They argue that market frictions that make leverage relevant may also be associated with a firm’s source of capital. We apply this intuition to our work, and divide firms on the basis of access to source of financing.

³Research that documents an increase in firm specific volatility includes Campbell, Lettau, Malkiel, and Xu (2001) for stock returns, and Comin and Mulani (2006) for real variables (sales, employment, investment). Philippon (2003), Comin and Philippon (2005) and Comin and Philippon (2005) suggest the competition hypothesis – that these divergent trends in macro and micro volatility can be explained by competitive pressures in the goods market that increase idiosyncratic risk for the firms but reduce the impact of demand shocks on the economy by 40%. Countercyclical markups models (see Rotemberg and Woodford (1999) for a review of these models) in general predict diverging trends for macro and firm volatilities as long as the elasticity of the markup with respect to aggregate shocks decreases with the degree of competition.
can obtain financing from banks or public debt markets. As we know, banks and private lenders are able to reduce adverse selection by obtaining private information about firms, and are also able to reduce moral hazard by better monitoring the firms.\textsuperscript{4} This allows banks to provide ‘informed’ debt to firms, by taking idiosyncratic risks of the specific firm into account.\textsuperscript{5} On the other hand, financing decisions by public debt markets are based on expectations about aggregate volatility, since the investors in the public debt markets can better diversify their exposure to individual firm-specific risks and public debt markets provide ‘uninformed’ arm’s length financing.

Why would we expect firm leverage to react negatively to increasing idiosyncratic volatility? Increased volatility increases the probability of a positive productivity shock. A firm which experiences a positive productivity shock would like to increase its capital stock and produce more in that period. However, the increase in capital stock must be financed either through internal cash flow, debt financing or by relatively costly equity issuance. When productivity volatility increases, the firm will want to make larger investments in some periods, however, at the same time it tries to minimize the amount of equity issuance. Thus, it becomes more important for a firm to have spare debt capacity to obtain additional financing, should the right conditions arise. DeAngelo, DeAngelo, and Whited (2011) illustrate this effect in a dynamic structural model. This mechanism which requires firms to respond to increase in idiosyncratic volatility by reducing leverage, is more applicable to firms that obtain financing from banks. Since investors who finance firms through public debt markets are able to diversify away the idiosyncratic risk of a firm, firms that obtain public debt financing

\textsuperscript{5}This is a reasonable assumption as long as banks are unable to completely diversify away the idiosyncratic risks of the borrowing firms through reselling almost all of the bank debt. FDIC Call Report Data for the last two decades show that the average bank holds approximately 10% of its assets as commercial and industrial loans, supporting the assumption that they are not reselling all the loans.
are less credit constrained and can borrow at a lower cost.

Our mechanism offers a couple of testable implications, which we show to be true in Section II: (i) The cost of financing should be more sensitive to firm specific uncertainty for firms obtaining bank debt than for firms that obtain debt from public debt markets. Indeed, we find that there is a strong correlation between idiosyncratic volatility and the cost of financing for unrated firms but a lower correlation between idiosyncratic volatility and the cost of financing for rated firms. (ii) The cost of financing should be more sensitive to aggregate uncertainty for firms obtaining public market debt than for firms obtaining debt from banks. Again, we find that the interest rates paid by rated firms are strongly correlated with aggregate volatility whereas the statistical relationship between aggregate volatility and the cost of financing for unrated firms is insignificant.

Finally, to quantitatively assess the impact of volatility on firm leverage over time, we develop a structural model of firm investment and capital structure choices in the presence of uncertainty. Our model is based on Hennessy and Whited (2007) and DeAngelo, DeAngelo, and Whited (2011). Firm’s profits depends on an idiosyncratic productivity shock which has a different dynamic for rated and unrated firms. The tightness of the borrowing constraints is allowed to be different in the two groups. This is intuitive because unrated firms generally face larger information asymmetry with capital markets, and experience tighter borrowing constraints. We estimate the model, including the parameters affecting the borrowing constraints of a firm in presence of idiosyncratic volatility obtained from U.S. data. The estimated model has a 16% lower borrowing capacity for unrated firms compared to rated firms. That difference along with changes in volatility of idiosyncratic productivity shocks is sufficient to explain the observed dynamics of leverage for unrated firms in our model. In other words, we find that the changes in volatility over the past 40 years can
explain almost all of the reduction in firm leverage for unrated firms.

The decrease in firm leverage and cash holdings over time is also documented by Armenter and Hnatkovska (2011) and Bates, Kahle, and Stulz (2009). Armenter and Hnatkovska (2011) attribute the fall in leverage to a decrease in the tax on dividends. However, this explanation is not consistent with the fact that the increase in cash holdings is driven by firms which do not pay dividends, as documented by Bates, Kahle, and Stulz (2009). It also would not explain why the decrease in leverage is driven by unrated firms, which we document. Bates, Kahle, and Stulz (2009), using a reduced form approach, find that an increase in idiosyncratic volatility is an important reason for the increase in cash holdings. They also find that a decrease in inventories, reduced capital expenditure and increased R&D\textsuperscript{6} intensity plays a role. In our paper, we build a structural model of firm capital structure choices under uncertainty. Using the variation in borrowing costs and borrowing limits we can also explain that these changes in capital structure are driven by unrated firms.

The rest of the paper is structured as follows: In Section II we use data on U.S. firms to study trends in leverage over time, as well as relationships between leverage, financial rating, idiosyncratic volatility and interest expenses. Section III builds a dynamic structural model of firm investment and capital structure choices under uncertainty. Section IV discusses the model calibration and presents our quantitative results. Section V concludes.

II Stylized Facts and Empirical Analysis

In this Section we present our empirical findings. We first document trends in leverage, idiosyncratic volatility and the volatility of interest expenses for US firms. We show that over the past 40 years there is a sharp decline in the leverage of US firms. This decline is,

\textsuperscript{6}R&D intensive firms hold more cash.
however, exclusively driven by firms that lack financial rating. At the same time as leverage is falling, we observe an increasing trend in the idiosyncratic volatility of all firms, a decline in aggregate volatility, an increase in the interest rate volatility of unrated firms and a decrease in the interest volatility of rated firms. After documenting these stylized facts, we turn to an investigation of whether the changes in uncertainty and can explain the trends in leverage.

**Data Sources**

We utilize CRSP-Compustat and LPC’s Dealscan database for this work. We obtain firm-year level financial data from Compustat. The Dealscan database consists of private loans made by bank and non-bank lenders to U.S. corporations during the period 1981–2010. The basic unit of loans reported in Dealscan is a loan facility. Loan facilities are grouped into packages. Packages may contain various types of loan facilities for the borrower. Loan information such as loan amount, maturity, type of loan, purpose of obtaining the loan and other information, is reported at the facility level. As discussed in Chava and Roberts (2008), Dealscan database contains between 50% to 70% of all commercial loans in the U.S. during the early 1990s. From 1995 onwards, Dealscan coverage increases to include an even greater fraction of commercial loans. Table III reports the summary statistics of the dataset composed of U.S. firms since 1970.

Following Faulkender and Petersen (2006) and Lemmon and Zender (2010), we predict the likelihood of a firm having access to public debt markets. We use the model to backfill ratings for firms prior to 1985, since our dataset is sparse before this point.
Stylized Facts

Figure 1 shows trends in book leverage for all firms during the sample period. We observe that leverage is pro-cyclical, however there is a strong declining trend through the whole sample period. Over the sample period, the trend line falls by about 45%, from 0.22 to 0.12. The decline in leverage is, however, driven by firms, which lack financial rating. In Figure 2 we plot the leverage of rated and unrated firms separately. There is a strong negative trend in leverage for unrated firms whereas the trend for rated firms is almost flat (weakly negative but statistically insignificant). The trend in leverage for unrated firms falls by about 75% throughout the sample period.

The increase in firm specific volatility manifests itself in various firm characteristics. (See Campbell, Lettau, Malkiel, and Xu (2001) for stock returns, and Comin and Mulani (2006) for real variables (sales, employment, investment).) Here we focus on the idiosyncratic productivity of firms. We obtain firm idiosyncratic productivity of firms on a yearly basis through a regression analysis on COMPUSTAT data, under the assumption that firm profits is given by a Cobb-Douglas function. We then follow Comin and Philippon (2005) in constructing the time path for the volatility of productivity shocks (in logs).

Figure 3, from Comin and Philippon (2005), shows the increasing trend in idiosyncratic volatility (in their case individual firm sales volatility) together with the falling trend in aggregate volatility (using GDP volatility). The fall in aggregate volatility, also known as “the great moderation” is a well known fact in macroeconomics and for instance documented by Kim and Nelson (1999) and McConnell and Perez-Quiros (2000). At a first glance it may appear contradictory with diverging trends in aggregate and idiosyncratic volatility. Comin and Mulani (2006), however, explain this with decreasing correlation between the productivity of sectors and firms. In Figure 4, we plot the trends in idiosyncratic volatility
of rated and unrated firms separately. It is clear that the volatility of both types of firms have been increasing during the sample period.

![Graph showing 10-Year Centered Rolling Standard Deviation of Growth Rates](image)

**Figure 3: Aggregate Volatility and Idiosyncratic Volatility of Firms**

Figure 5 displays the average interest expense paid by rated and unrated firms over the sample period, and Figure 6 displays the volatility of interest expenses. As can be seen from the figures there are no clear, strong time-trends in the level of interest expenses. The trend in the volatility of interest expenses for unrated firms does however, increase by about 40% over the sample period whereas the trend in the volatility of interest expenses for rated firms drop by about 30% over the same period. We relate this diverging pattern to the diverging trends in aggregate and idiosyncratic volatility. Unrated firms have to borrow from banks, which closely monitor the prospects of the individual firm. Because the idiosyncratic volatility of firms has increased, the interest rates faced by unrated firms have

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7The technical details behind these figures are described in the appendix.
also became more volatile. Rated firms on the other hand, borrow from public debt markets. We argue that their interest expenses is more closely tied to macroeconomic conditions. As macroeconomic volatility has decreased so has the volatility of the interest rates faced by rated firms.

**Sensitivity of interest rates to idiosyncratic and aggregate volatility**

To measure the impact of uncertainty on firm financing policy, we recognize that all firms are not equally affected by aggregate and idiosyncratic uncertainty. Access to financing is one of the differentiating factors that affects a firm’s exposure to risk. Public debt market financing is at arm’s length, i.e. public debt markets do not have private information about firms. Compared to private debt from banks who possess soft information about a firm, public debt financing is thus more dependent on the aggregate economy. Similarly, bank financing should be more sensitive to firm specific shocks since banks are able to obtain
private information about the firm.

Table I provides support to our argument. The table reports the sensitivity of firm
interest rates spreads to idiosyncratic and aggregate volatility. Idiosyncratic volatility is calculated as the cross sectional standard deviation in earnings for firms in each sector every year based on access to public markets. Aggregate volatility is the cross sectional standard deviation in earnings for all firms per year. Firm interest rate spreads is the AllinDrawn variable obtained from the Dealscan database for new loans initiated every year. Table I shows that the impact of idiosyncratic standard deviation is lower (-87.68 basis points in Column(6)) for firms with ratings. On the other hand, firms that have access to public debt markets have a higher loading on aggregate risk (78.75 basis points in Column(6)) of the economy as measured by the aggregate standard deviation.

III Model

Economic Environment

Our model is based on DeAngelo, DeAngelo, and Whited (2011) and Hennessy and Whited (2007). There is a continuum of infinitely-lived firms. To produce output (which generates operating profit), firms use a single input – capital. The firm’s operating profit ($\pi$) depends on the firm’s capital ($k$) and an idiosyncratic productivity shock ($z$). The firm finances its investments into productive capital by issuing one-period debt, equity, or using their own internal savings (“cash”). Equity owners are the residual claimants of the firm. They receive as dividends the portion of the firm’s after-tax revenues which are not spent on either financing new investment, paying the interest on the firm’s debt or increasing the firm’s internal savings. Similar to DeAngelo, DeAngelo, and Whited (2011) and Hennessy and Whited (2007), we do not restrict dividends to be non-negative and interpret negative dividends as new equity issuance.
Table I: Sensitivity of Firm Interest Rates to Idiosyncratic and Aggregate Volatility

The table reports the sensitivity of firm interest rates spreads to idiosyncratic and aggregate volatility. Idiosyncratic volatility is calculated as the cross sectional standard deviation in earnings for firms in each sector every year based on access to public markets. Aggregate volatility is the cross sectional standard deviation in earnings for all firms per year. Firm interest rate spreads is the *AllinDrawn* variable obtained from the Dealscan database for new loans initiated every year.

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<td>-7.599</td>
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<td>Idiosyncratic Std. Dev.</td>
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<tr>
<td>Year FE?</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
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Standard errors in parentheses, clustered by year
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
The task of the firm’s managers is to decide on the firm’s investment in productive capital and its financial policy to maximize the value of the firm at time 0 \( V_{t_0} \) which is equal to the expected present discounted value of today’s and future dividends to the firm’s shareholders:

\[
V_{t_0} = \max E_{t_0} \left[ \sum_{t=t_0}^{\infty} \frac{1}{(1 + \rho)^{t-t_0}} e_t \right]
\]

where \( \rho \) is the discount rate used by the firm’s shareholders, and \( e_t \) is the net cash flow (dividends paid) to the shareholders in period \( t \).

**Public Debt and Bank Debt**

Motivated by our findings in Section II, we assume that firms are divided into two groups. The firms in the first group, rated firms, have access to public debt markets, where they can issue debt, \( p \), by selling bonds. The firms in the second group, unrated firms, can only borrow from banks, and do not have access to public debt markets. This division of firms into the two groups is exogenous and fixed over time. In the model, the differences between the firms in the two groups are (i) the tightness of their borrowing constraints, and (ii) the interest rate at which they can issue their debt.

We use, \( \bar{p}_{pub} \) and \( \bar{p}_{bank} \) to denote the borrowing limits of firms in the two groups (“public” and “banking”) and \( r_{pub} \) and \( r_{bank} \) to denote the interest rate at which firms in the two groups can borrow. As demonstrated in Section II, the interest rate on publicly-traded debt is more positively correlated with the aggregate productivity shocks, while the interest rate on bank debt is more positively correlated with the firm’s idiosyncratic shocks.
**Equity Issuance Costs and Cash Holdings**

Equity issuance incurs exogenously given costs. Equity issuance costs can be interpreted as flotation or adverse selection costs, as in Myers and Majluf (1984). We use the linear-quadratic equity issuance cost function $\phi(e)$ that takes the following form:

$$
\phi(e) \equiv \Phi_e \left( \lambda_1 e - \frac{1}{2} \lambda_2 e^2 \right)
$$

where $\lambda_i \geq 0, \quad i = 1, 2.$

$$
\Phi_e = 1_{e<0}.
$$

The cost of issuance is positive, increasing and concave (i.e. $\lambda_1 - \lambda_2 e > 0$), and is incurred when $e < 0$. This is because indicator function $\Phi_e = 1$ when $e < 0$.

We interpret negative level of debt as cash holdings. Firms earn interest rate $r$ on their cash balances on which they pay tax $\tau_c$. Keeping a cash balance allows the firm to finance projects internally, but comes at an immediate cost, which can be motivated by differential borrowing and lending rates and “agency costs”. Fraction $1 - \eta$ of the cash balances are incurred as cost in every period. As in DeAngelo, DeAngelo, and Whited (2011), the cost of holding cash is given by:

$$
(1 - \eta)p(1 + r(1 - \tau_c))\Phi_p, \quad \Phi_p = 1_{p<0}.
$$

The cost is only present when $p < 0$, i.e. cash balances are positive (given by indicator variable $\Phi_p = 1$ when $p < 0$).
Firm Profits and Taxes

The firm’s operating profit $\pi(z, k)$ depends on the firm’s idiosyncratic productivity shock, $z$, and capital, $k$. Corporate taxable income $y$ is equal to operating profit less economic depreciation which occurs at rate $\delta$) less interest expense and interest income:

$$y = \pi(z^i, k) - \delta k - rp.$$  

We assume that the corporate tax rate $\tau_c$ is flat, so that the firm pays the following amount as tax:

$$\tau_c (\pi(z^i, k) - \delta k - rp)$$

Capital Adjustment Costs

The firm can choose to invest in productive capital after paying an adjustment cost. Invested capital depreciates at rate $\delta$. The firm incurs capital stock adjustment costs that take the following linear-quadratic form:

$$A(k, k') = \Phi_i \left( \gamma k + \frac{a}{2} \left( \frac{k' - (1 - \delta)k}{k} \right)^2 \right)$$

Indicator function $\Phi_i = 1$ whenever the firm invests. The first term captures the fixed cost of investment per unit of capital $k$ given by $\gamma$. The second term captures convex adjustment costs of level of capital compared to the depreciated capital of last period. $a$ is a constant.
The Problem of the Firm

The firm starts the period with capital $k$ and debt $p$ chosen in the previous period, and the interest rate, $r$, determined in the previous period, that it has to pay on its debt, $p$ (or receive as an income if $p < 0$). As we mentioned above, if $p$ is negative, we interpret it as cash holdings. The firm chooses new capital $k'$ and debt level $p'$ subject to the borrowing limit $p' \leq \bar{p}$.

Using the firm’s budget constraint, the net cash flow generated by the firm before the payment of the equity issuance costs is:

$$e = (1 - \tau_c) \pi(z, k) + \tau_c \delta k + \left[ p' - p(1 + r(1 - \tau_c))(1 - \eta \Phi_p) \right] - \left[ (k' - (1 - \delta)k) + A(k, k') \right]$$  \hfill (2)

In other words, the sources of cash flow of the firm are after tax profits (first term), tax shield benefits of depreciation (second term) and net issuance of debt (third term) including benefits from holding cash; the uses of financing of the firm are to finance new investments (fourth term) including adjustment costs.

To write the problem of the firm recursively, we define $\tilde{p} = p(1 + r(1 - \tau_c))$ and rewrite the net cash flow equation as$^8$:

$$e = (1 - \tau_c) \pi(z, k) + \tau_c \delta k + \left[ \frac{\tilde{p}'}{1 + r'(1 - \tau_c)} - \tilde{p}(1 - \eta \Phi_{\tilde{p}}) \right] - \left[ (k' - (1 - \delta)k) + A(k, k') \right]$$  \hfill (3)

where $r'$ is the interest rate that the firm will have to pay in the next period$^9$.

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$^8$Note that $\Phi_p = \Phi_{\tilde{p}}$.

$^9$Using this transformation allows us to “save” one dimension of the state space – we do not need to keep track of $r$, the interest rate determined in the previous period which we have to pay in the current period on the outstanding debt.
We can express the firm’s problem recursively as follows:

\[
V(k, \tilde{p}, z) = \max_{k', \tilde{p}'} \left( e(k, k', \tilde{p}, \tilde{p}', z) + \phi(e(k, k', \tilde{p}, \tilde{p}', z)) + \frac{1}{1 + \rho} \int V(k', \tilde{p}', z') d\Gamma(z, z') \right),
\]

the recursive problem of the firm is to choose capital \( k \) and debt \( p \) in the next period to maximize the infinite discounted stream of cash flows net of costs of financing and investment given the present state variables (capital \( k \), debt \( p \), idiosyncratic shock \( z \)).

**First-order conditions**

First-order condition with respect to capital in the next period \( k' \) yields:

\[
\phi'_{e,t} + 1 \left( 1 + a \frac{k_{t+1} - (1 - \delta)k_t}{k_t} \right) = \phi'_{e,t+1} + 1 \left( (1 - \tau_c) \alpha \zeta_{t+1} k_{t+1}^{\alpha - 1} + (1 - \delta + \tau_c \delta) - \Phi \gamma + \frac{a (k_{t+2} - (1 - \delta)k_{t+1})^2}{k_{t+1}^2} \right).
\]

where \( \phi'_{e,t} \) is the marginal cost of issuing equity in period \( t \), Expected Future Marginal Adjustment Cost represents the effect of current choice on capital adjustment on the future adjustment costs faced by the firm:

\[
\phi'_{e} = \Phi_e (\lambda_1 - \lambda_2 e)
\]

First-order condition with respect to future debt \( \tilde{p}' \) yields, an euler equation in terms of equivalence of obtaining one more dollar of debt and the payment that will entail in the
future over all states of the world:

\[
(\phi_{e,t} + 1) = \frac{1 + r_{t+1}(1 - \tau_{c})}{1 + \rho} E_{t} \left( \phi_{e,t+1} + 1 \right) \left( 1 - \eta \Phi_{p_{t+1}} \right).
\]  

(6)

IV Calibration and Results

Following Hennessy and Whited (2007), we choose the following functional form for firm’s operating profit \( \pi \):

\[
\pi(z^{i}, k) = z^{i} k^{\alpha},
\]

where \( \alpha \in (0, 1) \). We assume that the idiosyncratic productivity shock follows an AR(1) process:

\[
\ln(z^{i}_{t}) = \rho^{j} \ln(z^{i}_{t-1}) + \varepsilon_{i}
\]

where \( \varepsilon_{i} \sim N(0, \sigma^{j}) \), with \( j \in \{r, u\} \) denoting the group of firms (rated or unrated).

Changes in volatility of firm-level productivity

As mentioned previously, Comin and Mulani (2006) document the increase in firm-level volatility in the growth rate of sales. However, in our model, we will use the volatility of the firm-level productivity as an input into our model, while firms’ sales and operating profits will be determined endogenously. Thus, we need to document what happened to firm-level volatility of productivity shocks. To measure firm-level volatility of productivity shocks, we used the same methodology as Comin and Mulani (2006) and Comin and Philippon (2005), combined with the assumption that firm production function has the Cobb-Douglas functional form, with capital being the only production input.
We use Earning before interest and taxes (EBIT), deflated using PPI, as a measure of firms’ output \( (y_i) \), and the sum of Property, plant and equipment: Buildings at cost (FATB) and Property, plant and equipment: Machinery and equipment at cost (FATE), also deflated using PPI, as a measure of firm’s capital input \( k_i \). Next, we compute the logarithm of firm-level productivity shock as a residual from the following regression:

\[
\log(y_i) = \beta_0 + \beta_1 \log(k_i) + \varepsilon_i, \tag{7}
\]

where we include year and industry level fixed effects.

Note that productivity shocks in levels are \( \text{TFP}_i = \exp(\varepsilon_i) \). Next, we follow Comin and Philippon (2005) and for each firm, compute the time path for the volatility of productivity shocks (in logs) as the time series of standard deviations of ten-years rolling windows of the productivity shocks constructed as above. Namely, we compute:

\[
\sigma(\log(\text{TFP}_{i,t})) = \sqrt{\frac{\sum_{\tau=t-4}^{t+5} (\varepsilon_{i,\tau} - \bar{\varepsilon}_{i,t})^2}{9}} \tag{8}
\]

Finally, for every period \( t \), we find the median \( \sigma(\log(\text{TFP}_{i,t})) \) over all firms, \( i \).

Figures 3 and 4 report the results. The median standard deviations of log-TFP shocks for all firms increases from 0.2930 in 1983 to 0.5012 in 2006. Among the subset of unrated firms, our measure of volatility increased from 0.3718 to 0.53, while for the subset of rated firms it increased from 0.2131 to 0.34 over the same period. This suggests the differences in the increase in volatility of productivity shocks among these two groups of firms cannot explain different trends in their leverage levels (if anything, such differences would suggest that the leverage levels for the rated firms should have decreased more).

For our calibration, we also need the autocorrelation of productivity shocks. Since in our
experiment, we will be mainly interested in changing the volatility of productivity shocks, we will assume that the autocorrelation of these shocks has remained the same over the period of study. To compute the autocorrelation of the log-TFP shocks, we use a procedure similar to the one above that we used to compute the time paths for the standard deviations, but with one major difference – instead of computing the statistics for each year using a time window around that given year, we use the autocorrelation for the whole time period once. We get that the autocorrelation of log-TFP shocks is 0.46667 for unrated firms, and 0.7597 for rated firms.

**Estimation**

Table II reports the six model parameters that are calibrated using an exactly identified simulated method of moments approach. Panel A reports the parameters for unrated firms and Panel B reports them for rated firms. We minimize the squared percent deviation of simulated moments from the six data moments also reported in Table II. Let \( \Theta \) denote a vector of the estimated parameters, and let \( V(\Theta) = [V_1(\Theta), \ldots V_6(\Theta)]' \) denote the vector where \( V_i(\Theta) \) denotes the percentage deviation between the data moment and simulated moment. Then, we attempt to minimize \( V(\Theta)'V(\Theta) \) by choosing parameter values.

We keep all data moments except leverage to be the same between rated and unrated firms. This ensures that we are only allowing the dimension (leverage) we seek to explain be different between rated and unrated firms. If we allow other firm characteristics to different, then intuitively we will be able to explain more of the variation in leverage dynamics of rated and unrated firms over time, however that explanation will depend upon differences in characteristics that are not the mechanism in question.

We allows the tightness of the borrowing constraint \( \bar{p} \) to be different in the two groups.
Table II: Model Fit

Moments are calculated using all firms in the dataset. Except leverage \(d/K\), which is obtained from the sub-sample of unrated firms in Panel A, and rated firms in Panel B.

### Panel A: Unrated firms

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average investment, (I/K)</td>
<td>0.1441</td>
<td>0.1796</td>
</tr>
<tr>
<td>Variance of investment, (I/K)</td>
<td>0.0860</td>
<td>0.1241</td>
</tr>
<tr>
<td>Leverage, (d/K), in 1970</td>
<td>0.1883</td>
<td>0.1861</td>
</tr>
<tr>
<td>Average equity issuance, (e/K)</td>
<td>0.0655</td>
<td>0.0238</td>
</tr>
<tr>
<td>Variance of equity issuance, (e/k)</td>
<td>0.0222</td>
<td>0.0255</td>
</tr>
<tr>
<td>Average cash balances, (c/K), conditional on (c &gt; 0)</td>
<td>0.1297</td>
<td>0.0357</td>
</tr>
</tbody>
</table>

### Panel B: Rated firms

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average investment, (I/K)</td>
<td>0.1441</td>
<td>0.1450</td>
</tr>
<tr>
<td>Variance of investment, (I/K)</td>
<td>0.0860</td>
<td>0.1034</td>
</tr>
<tr>
<td>Leverage, (d/K), in 1970</td>
<td>0.3317</td>
<td>0.3311</td>
</tr>
<tr>
<td>Average equity issuance, (e/K)</td>
<td>0.0655</td>
<td>0.0810</td>
</tr>
<tr>
<td>Variance of equity issuance, (e/k)</td>
<td>0.0222</td>
<td>0.0107</td>
</tr>
<tr>
<td>Average cash balances, (c/K), conditional on (c &gt; 0)</td>
<td>0.1297</td>
<td>0.0998</td>
</tr>
</tbody>
</table>
As discussed before, this is intuitive because theory suggests that unrated firms face larger information asymmetry with capital markets, and experience tighter borrowing constraints. The estimated model has a 16% lower borrowing capacity for unrated firms compared to rated firms, which confirms our prior.

Next, we find this difference along with changes in volatility of idiosyncratic productivity shocks to be sufficient to explain the observed dynamics of leverage for unrated firms in our model.

**Results and Future Work**

The main question we want to answer is what fraction of change in leverage can be explained by changes in idiosyncratic volatility of firms. To answer this question, we increase the volatility of TFP shock from 0.3718 to 0.53 in our model for unrated firms. The volatility numbers are obtained from the documented increase in idiosyncratic volatility during the last 40 years. The average leverage of firms in the model drops from 0.1861 to 0.0498 as a response to this increase in idiosyncratic volatility. Comparing our model generated decline in leverage for unrated firms with the data reported in Figure 2, we can see that the model is able to generate the strong negative trend in leverage for unrated firms. The trend in leverage for unrated firms falls by about 75% throughout the sample period in the data, and we are able to create a similar amount of decline.

For rated firms, in the same period, the volatility of TFP shocks increased from 0.2131 to 0.34. Feeding these numbers in our model, the average leverage of rated firms declines from 0.3317 to 0.232, which is about 30% reduction in leverage. The model slightly overestimates the trend for rated firms, i.e. rated firms did not engage in as much precautionary reduction in leverage as predicted by the model. Given our parsimonious approach where we have
restricted all parameters for unrated and rated firms to be the same except TFP volatility and borrowing limit, such limited difference between model generated moments and data is expected. The difference may be due to additional means of obtaining financing that are available to rated firms. In order to explain why rated firms did not reduce their leverage as much as predicted by the model, we are relaxing the assumption that all remaining firm characteristics are similar for rated and unrated firms. Another mechanism that we are in the process of exploiting is the change in interest rate volatility as shown in Figure 6.

We also plan to decompose the relative contribution of the two main differences between rated and unrated firms that we model – differences in borrowing limits and differences in idiosyncratic volatility of productivity shocks. The two mechanisms are quite different since increasing trends in idiosyncratic volatility in the economy are driven by competition and other exogenous factors which firms cannot control. On the other hand, differences in borrowing limits depend on firm level information asymmetry and can be ameliorated by changing financing policy at the firm level. Thus the normative implications of changing leverage may be quite different based on which of the explored mechanism contributes more to the decline in leverage for unrated firms.

V Conclusion

In this paper, we document that the observed decline in leverage over the last few decades is much more pronounced among firms that do not have access to public debt markets. We argue that most of the reduction in firm leverage for unrated firms can be explained through precautionary financing motives of firms, which are insuring themselves against increasing idiosyncratic volatility. We also show that unrated firms are more sensitive to idiosyncratic
volatility. This is intuitive because rated firms obtain financing from public debt markets, and since investors in public debt markets can better diversify their exposure to individual firm-specific risks, the sensitivity of rated firms to idiosyncratic volatility is lower.

Increased idiosyncratic volatility increases the probability of a productivity shock. A firm which experiences a positive productivity shock would like to increase its capital stock and hence would like to have more spare debt capacity to obtain additional financing as volatility increases. Our structural model of firm investment and capital structure choices in the presence of uncertainty shows that the changes in volatility over the past 40 years explain almost all of the reduction in firm leverage for unrated firms.
References


Table III: Summary Statistics
This table presents summary statistics for all non-financial and non-utility firms available in COMPUSTAT. The sample includes firm-year observations for the period 1970 – 2012. Cash represents cash scaled by assets, Interest expenditure is scaled by total debt, Maturity represents ratio of current liabilities to long term liabilities, Profitability is operating income before depreciation scaled by assets, Tangibility is net property, plant and equipment scaled by assets, Book Leverage is total debt scaled by assets and firm size is logarithm of assets in millions of dollars. Standard deviations is in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>All Firms</th>
<th>Rated</th>
<th>Unrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>0.153</td>
<td>0.0759</td>
<td>0.165</td>
</tr>
<tr>
<td></td>
<td>(0.200)</td>
<td>(0.0991)</td>
<td>(0.209)</td>
</tr>
<tr>
<td>Interest Expenditure</td>
<td>0.129</td>
<td>0.0903</td>
<td>0.136</td>
</tr>
<tr>
<td></td>
<td>(0.221)</td>
<td>(0.104)</td>
<td>(0.235)</td>
</tr>
<tr>
<td>Bank Loan Spread (bps)</td>
<td>215.2</td>
<td>180.0</td>
<td>240.8</td>
</tr>
<tr>
<td></td>
<td>(146.4)</td>
<td>(141.5)</td>
<td>(144.6)</td>
</tr>
<tr>
<td>Maturity</td>
<td>2.218</td>
<td>0.664</td>
<td>2.525</td>
</tr>
<tr>
<td></td>
<td>(9.134)</td>
<td>(5.013)</td>
<td>(9.713)</td>
</tr>
<tr>
<td>Profitability</td>
<td>0.0103</td>
<td>0.129</td>
<td>-0.00763</td>
</tr>
<tr>
<td></td>
<td>(0.422)</td>
<td>(0.0884)</td>
<td>(0.449)</td>
</tr>
<tr>
<td>Tangibility</td>
<td>0.319</td>
<td>0.372</td>
<td>0.311</td>
</tr>
<tr>
<td></td>
<td>(0.246)</td>
<td>(0.239)</td>
<td>(0.246)</td>
</tr>
<tr>
<td>Book Leverage</td>
<td>0.292</td>
<td>0.414</td>
<td>0.273</td>
</tr>
<tr>
<td></td>
<td>(0.325)</td>
<td>(0.272)</td>
<td>(0.328)</td>
</tr>
<tr>
<td>Firm Size (Log Assets)</td>
<td>4.319</td>
<td>7.778</td>
<td>3.796</td>
</tr>
<tr>
<td></td>
<td>(2.534)</td>
<td>(1.631)</td>
<td>(2.214)</td>
</tr>
<tr>
<td>Observations</td>
<td>334048</td>
<td>43882</td>
<td>290166</td>
</tr>
</tbody>
</table>